5

Image Evaluation Methods for PIV

This chapter treats the fundamental techniques for statistical PIV evaluation. In spite of the fact that most realizations of PIV evaluation systems are quite similar – in nearly every case they are based on digitally performed Fourier algorithms – we will also consider optical techniques because they are still important for the classification and understanding of existing setups.

In order to extract the displacement information from a PIV recording some sort of interrogation scheme is required. Initially, this interrogation was performed manually on selected images with relatively sparse seeding which allowed the tracking of individual particles [125, 178]. With computers and image processing becoming more commonplace in the laboratory environment it became possible to automate the interrogation process of the particle track images [118, 185, 231, 246]. However, the application of tracking methods, that is to follow the images of an individual tracer particle from exposure to exposure, is only practicable in the low image density case, see figure 1.5(a). The low image density case often appears in strongly three-dimensional high-speed flows (e.g. turbomachinery) where it is not possible to provide enough tracer particles or in two phase flows, where the transport of the particles themselves is investigated. Additionally, the Lagrangian motion of a fluid element can be determined by applying tracking methods [127, 162, 229].

In principle, however, a high data density is required on the PIV vector maps, especially for the comparison of experimental data with the results of numerical calculations. This demand requires a medium concentration of the images of the tracer particles in the PIV recording. (In particular, in air flows it is not possible to achieve a high image density, because beyond a certain level the number of detectable images cannot be increased by further increasing the tracers density in the flow [53].) Medium image concentration is characterized by the fact that matching pairs of particle images – due to subsequent illuminations – cannot be detected by visual inspection of the PIV recording, see figure 1.5(b). Hence statistical approaches, which will be described in the next sections, had to be developed. After a statistical evaluation has been performed first, tracking algorithms can be applied additionally in order to achieve subwindow spatial resolution of the measurement, which is known as *super resolution* PIV [179]. However, since the extraction of the displacement information from individual particle images requires spatially well resolved recordings of the particle images, those techniques are more appropriate to increase the spatial resolution of photographic PIV recordings than that of digital recordings.

Tracking algorithms have continuously been improved during the past decade. Methods like the application of neural networks [134, 250] seem to be very promising. Thus, for some applications particle tracking might be an interesting alternative to statistical PIV evaluation methods on which we focus in this book. Readers interested in obtaining a survey of tracking methods are referred to the survey paper by Grant [36], to the lecture notes on *Three-dimensional velocity and vorticity measuring and image analysis techniques*, edited by Th. Dracos [34] and to the section on low image density PIV in the SPIE milestone series on PIV [35].

A comparison between cross-correlation methods and particle tracking techniques together with an assessment of their performance has recently been performed in the framework of the *International PIV Challenge* [48, 50].

5.1 Correlation and Fourier Transform

5.1.1 Correlation

The main objective of the statistical evaluation of PIV recordings at medium image density is to determine the displacement between two patterns of particle images, which are stored as a 2D distribution of gray levels. Looking around in other areas of meteorology, it is common practice in signal analysis to determine, for example, the shift in time between two (nearly) identical time signals by means of correlation techniques. Details about the mathematical principles of the correlation technique, the basic relations for correlated and uncorrelated signals and the application of correlation techniques in the investigation of time signals can be found in many textbooks [2, 20]. The theory of correlation can be extended in a straight forward manner from the one dimensional (1D time signal) to the two-dimensional (2D gray value distribution) case [4]. In chapter 3 the use of auto- and cross-correlation techniques for statistical PIV evaluation has already been explained. Analogously to spectral time signal representations, a 2D spatial signal $I(x, y)$ the power spectrum $|\hat{I}(r_x, r_y)|^2$ can be determined where r_x, r_y are spatial frequencies in orthogonal directions. The basic theorems for correlation and Fourier transform known from the theory of time signals are also valid for the 2D case (with appropriate modifications) [4].

For the calculation of the autocorrelation function two possibilities exist: either direct numerical calculation or indirectly (numerically or optically), using the Wiener-Kinchin theorem [2, 4]. This theorem states that

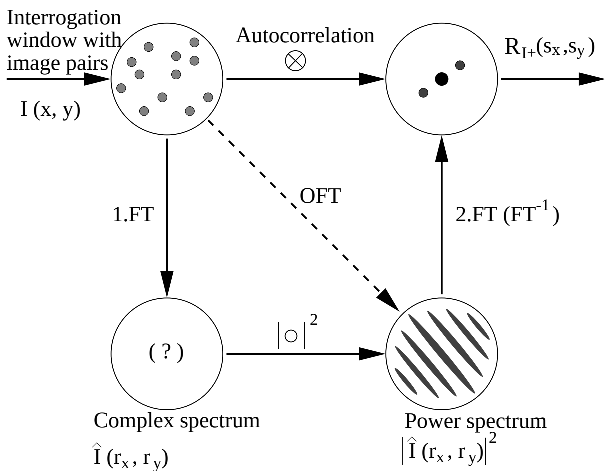

Fig. 5.1. Sketch of relation between 2D correlation function and spatial spectrum by means of the Wiener–Khinchin theorem. FT – Fourier transform, FT^{-1} – inverse Fourier transform, OFT – optical Fourier transform.

the Fourier transform of the autocorrelation function R_{I} and the power spectrum $|\hat{I}(r_x, r_y)|^2$ of an intensity field $I(x, y)$ are Fourier transforms of each other.

Figure 5.1 illustrates that the autocorrelation function can either be determined directly in the spatial domain (upper half of the figure) or indirectly by Fourier transform FT (left hand side), multiplication, that is the calculation of the squared modulus, in the frequency plane (lower half of the figure), and by inverse Fourier transform FT^{-1} (right hand side).

5.1.2 Optical Fourier Transform

As already mentioned in section 2.6 the far field diffraction pattern of an aperture transmissivity distribution is represented by its Fourier transform [10, 18, 102]. A lens can be used to transfer the image from the far field close to the aperture. For a mathematical derivation of this result some assumptions have to be made, which are described by the Fraunhofer approximation. These assumptions (large distance between object and image plane, phase factors) can be fulfilled in practical optical setups for Fourier transforms.

Figure 5.2 shows two different setups for such optical Fourier processors. In the arrangement on the left hand side the object, which would consist of a transparency to be Fourier transformed (e.g. the photographic PIV recording), is placed in front of the so-called Fourier lens. In the second setup (right hand side) the object is placed behind the lens. As derived in the book of Goodman [10] both arrangements differ only by the phase factors of the complex spectrum and a scale factor. Light sensors (photographic plates as

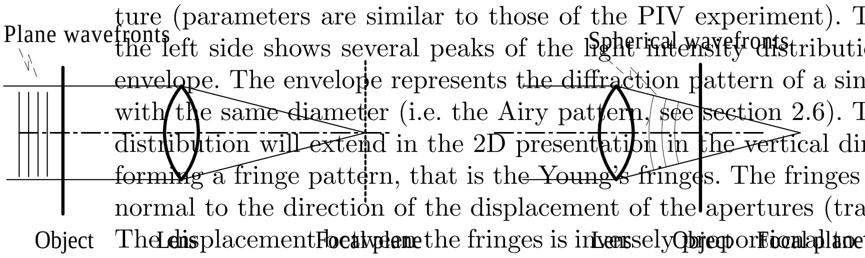

Fig. 5.2. Optical Fourier processor, different positions of object and Fourier lens.

well as CCD sensors) are only sensitive to the light intensity. The intensity corresponds to the squared modulus of the complex distribution of the electromagnetic field; hence phase differences in the light wave cannot be detected. Therefore, both of the arrangements shown in figure 5.2 can be used for PIV evaluation. The result of the optical Fourier transform (OFT, dashed line in figure 5.1) directly is the power spectrum of the gray value distribution of the transparency.

In the following this will be illustrated for the case of a pair of two particle images. White (transparent) images of a tracer particle on a black (opaque) background will form a double aperture on the photographic PIV recording. With good lens systems the diameter of an image of a tracer particle on the recording is of the order of 20 to 30 μm. The spacing between the two images of a tracer particle should be approximately 150–250 μm, in order to obtain optimum conditions for optical evaluation (compare section 4.3). Figure 5.3 shows a cross-sectional cut through the diffraction pattern of a double aperture (parameters are similar to those of the PIV experiment). The figure at the left side shows several peaks of the light intensity distribution under an envelope. The envelope represents the diffraction pattern of a single aperture with the same diameter (i.e. the Airy pattern, see section 2.6). The intensity distribution will extend in the 2D presentation in the vertical direction, thus forming a fringe pattern, that is the Young's fringes. The fringes are oriented normal to the direction of the displacement of the apertures (tracer images). The displacement between the fringes is inversely proportional to the displacement of the apertures (tracer images). If the distance between the apertures (tracer images) is decreased, the distance between the fringes will increase inversely. This is illustrated in the center of figure 5.3, where the distance between the two apertures is only half that of the example on the left side. It can be seen that the distance between the fringes is increased by a factor of two. The same inverse relation, which is due to the scaling theorem of the Fourier transform, is valid for the envelope of the diffraction pattern: if the diameter of the aperture (particle images) decreases, the extension of the Airy pattern will increase inversely (see figure 5.3, right side). As a consequence, more fringes can be detected in those fringe patterns which are generated by

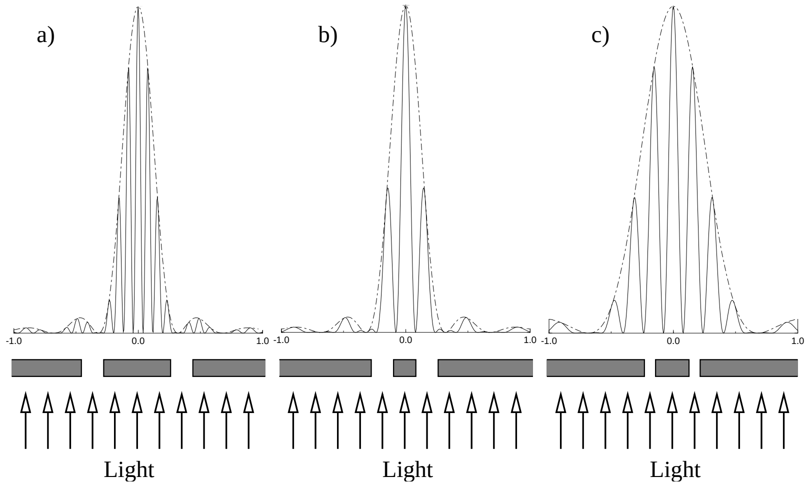

Fig. 5.3. Fraunhofer diffraction pattern of three different double apertures, from left to right, first the separation between the apertures has been decreased, then – on the right hand side – the diameter of the apertures has been decreased.

smaller apertures (particle images). This is one reason to explain why small and well focused particle images will increase the quality and detection probability in the evaluation of PIV recordings. Due to another property of the Fourier transform, that is the shift theorem, the characteristic shape of the intensity pattern does not change if the position of the particle image pairs is changed inside the interrogation spot. Increasing the number of particle image pairs also does not change the Young's fringe pattern significantly. Of course this is not true for the case of just two image pairs: two fringe systems of equal intensity will overlap, allowing no unambiguous evaluation.

5.1.3 Digital Fourier Transform

The digital Fourier transform is the basic tool of modern signal and image processing. A number of textbooks describe the details [2, 4, 15, 29]. The breakthrough of the digital Fourier transform is due to the development of fast digital computers and to the development of efficient algorithms for its calculation (Fast Fourier Transformation, FFT) [2, 4, 5, 29]. Those aspects of the digital Fourier transform relevant for the understanding of digital PIV evaluation will be described in section 5.4.

5.2 Summary of PIV Evaluation Methods

In the following the different methods for the evaluation of PIV recordings by means of correlation and Fourier techniques will be summarized.

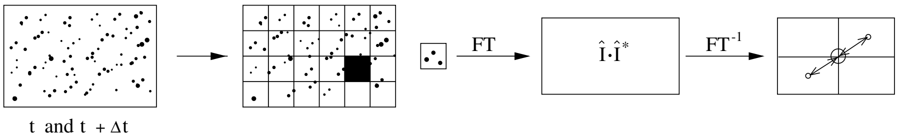

Fig. 5.4. Analysis of single frame/double exposure recordings: the fully digital autocorrelation method.

Figure 5.4 presents a flow chart of the fully digital autocorrelation method, which can be implemented in a straight forward manner following the equations given in chapter 3. The PIV recording is sampled with comparatively small interrogation windows (typically 20 - 50 samples in each dimension). For each window the autocorrelation function is calculated and the position of the displacement peak is determined. The calculation of the autocorrelation function is carried out either in the spatial domain (upper part of figure 5.1) or – in most cases – via the bypass over the frequency plane through the use of FFT algorithms.

If the PIV recording system allows the employment of the double frame/-single exposure recording technique (see figure 4.2) the evaluation of the PIV recordings is performed by cross-correlation (figure 5.5). In this case, the cross-correlation between two interrogation windows sampled from the two recordings is calculated. As will be explained later in section 5.4, it is advantageous to offset both these samples according to the mean displacement of the tracer particles between the two illuminations. This reduces the in-plane loss of correlation and therefore increases the correlation peak strength. The calculation of the cross-correlation function is generally computed numerically by means of efficient FFT algorithms.

Single frame/double exposure recordings may also be evaluated by a cross-correlation approach instead of autocorrelation (figure 5.6). In this case the interrogation windows can be chosen of different size and/or slightly displaced with respect to each other in order to compensate for the in-plane loss of correlation due to the mean displacement of particle images. Depending on the different parameters, autocorrelation peaks may also appear in the correlation plane in addition to the cross-correlation peak. This is illustrated in more detail in figure 5.6.

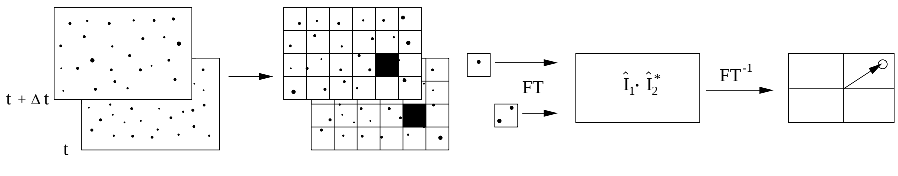

Fig. 5.5. Analysis of double frame/single exposure recordings: the digital cross-correlation method.

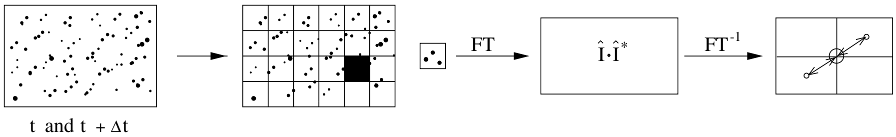

Fig. 5.6. Single frame/double exposure cross-correlation method flow chart.

The counterpart of the fully digital evaluation by means of autocorrelation is a system employing optical Fourier transform (OFT) for evaluation. In order to obtain the autocorrelation function a setup with two optical Fourier processors has to be implemented, following the bypass through the frequency plane as outlined in figure 5.1. A spatial light modulator is required to store the output of the first Fourier processor and to serve as input of the second Fourier processor. This is shown in figure 5.7. Up to now, no optical setups giving the 2D cross-correlation function for PIV evaluation have been described in literature.

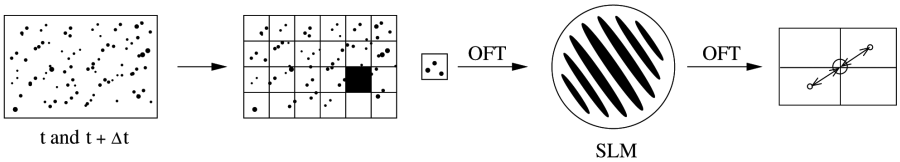

Fig. 5.7. Analysis of single frame/double exposure recordings: the fully optical method.

Computer memory and computation speed being limited in the beginning of the eighties, PIV work was strongly promoted by the existence of optical evaluation methods. The most widely used method was the Young's fringes method, which in fact is an optical-digital method, employing optical as well as digital Fourier transforms for the calculation of the correlation function. The flow chart of this evaluation method is shown in figure 5.8.

In the next section the fully optical method of PIV evaluation will be treated in order to give an introduction to the problems of this "old-fashioned" technique which still offers some advantages compared to digital techniques.

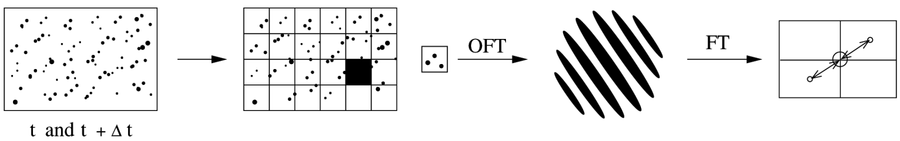

Fig. 5.8. Analysis of single frame/double exposure recordings: the hybrid (optical/digital) method utilizing the Young's fringes technique.

The most commonly used and very flexible digital evaluation methods will be discussed in the sections thereafter in more detail.

5.3 Optical PIV Evaluation

In order to achieve high quality in optical PIV evaluation some pre-processing of the recordings is required.

Due to the granular composition of the emulsion, photographic noise is contained in every photographic recording in addition to the particle images. This noise hampers the classical optical/digital evaluation of photographic PIV recordings. The noise is generated by scattering of the light from the illuminating laser beam at the film grain (grain noise) and variations of the refractive index (phase noise). Principally, phase noise can be reduced by immersing the negative in an index matching liquid as reported by Pickering & Halliwell [122]. However, a much better improvement of the Young's fringes visibility and, thus, of the probability of detecting valid velocity data during PIV evaluation can be achieved by a two step photographic process [123]. Interrogating the original PIV negative (park images of tracer particleson a bright background), the noise in the Fourier plane where the Young's fringes are formed reaches a considerable level because the areas on the negative which have the highest transmittance (background) maintain the gross fog (figure 2.44). By preparing a contact copy from the negative a positive transparency can be obtained (i.e. bright images of tracer particles on a dark background) which reduces the bias transmittance. This process prevents noise being transferred to the contact copy, which will be employed for evaluation, by taking advantage of the nonlinear behavior of the film used for copying. Thus, a much better signal-to-noise ratio can be obtained during PIV evaluation especially in regions of the PIV recording where the image density is low.

5.3.1 Young's Fringes Method

An experimental setup for the implementation of the Young's fringes technique is shown in figure 5.9. In this setup only the first Fourier transform (compare figure 5.8) is performed optically.

In order to determine the local autocorrelation, the input to the first optical Fourier processor is achieved by simply illuminating a small area (i.e. the interrogation spot) of the photographic negative of the PIV exposure with a He-Ne laser light beam. After optical Fourier transform by means of an arrangement already shown on the right side of figure 5.2, the Young's fringe pattern is obtained in the Fourier plane. The light intensity distribution in the Fourier plane is recorded by means of a video camera. Its image is digitized and stored in a computer. As explained, the spacing of the fringes is

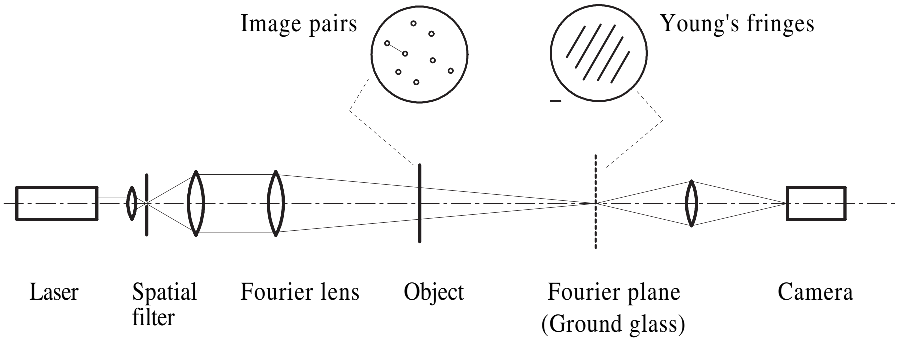

Fig. 5.9. Setup for PIV evaluation employing the Young's fringes technique.

inversely proportional to the displacement of matched image pairs. The direction of the fringes is perpendicular to the direction of the displacement. Thus, by evaluating the distance between the fringes and their direction, the magnitude and velocity of the tracer particles in the flow can be determined. The second better and more widely used method to evaluate the Young's fringes pattern is to perform a second Fourier transform by means of the FFT algorithm and the peak is found numerically in the computer. The major advantage of this procedure was the increased speed (in the nineteen-eighties) and the higher accuracy of the optical-digital method as compared to digital-digital methods. Setups like the one described here have been widely used in the first ten years of the development of PIV. Details about the different optical evaluation techniques for PIV and the Young's fringes method can be found in [37].

The problem in the development of fully optical evaluation systems was to store the output of the first optical processor (i.e. the Young's fringes) in such a way that it can be used as input to a second optical Fourier processor. Only after the development of easy to use and cheap spatial light modulators (SLM's) it was possible to set up fully operational optical PIV evaluation systems. For the details of fully optical PIV evaluation methods and their experimental realization, the interested reader might refer to [10, 114, 116, 117, 120, 121, 124, 168].